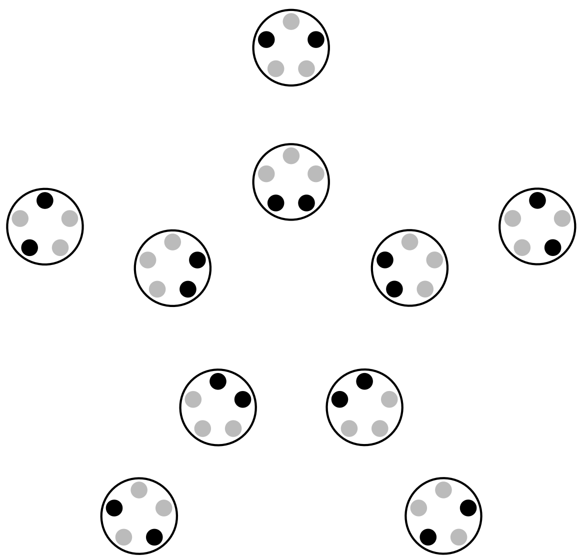
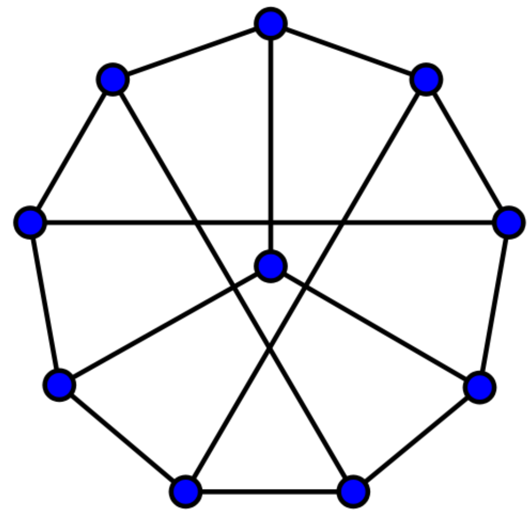
1 Constructing a graph from inter-related objects

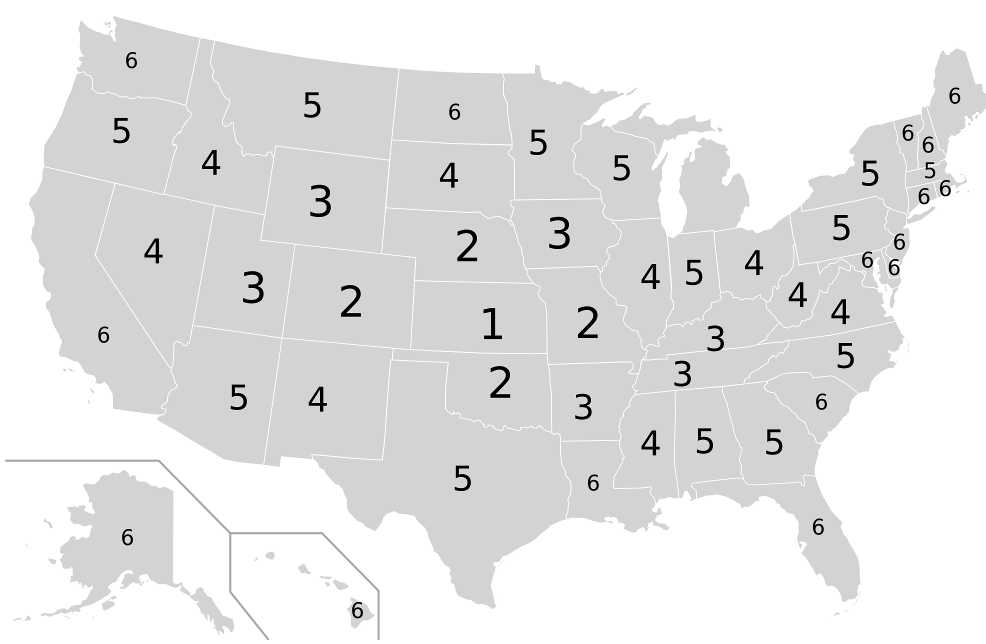
1. Add an edge between each pair of sets that don't intersect.
2. How many neighbours does each vertex of the graph have? Can you explain why?
3. Can you show that the graph on the left is "the same" as the graph on the right? How would you do this?





2 Map colouring: A problem on planar graphs

1. Colour the map using 4 colours, so that no neighbouring states have the same colour. Start with the states numbered 1, then 2, and so on. It should be easy this way.
2. Look at Utah and its five neighbours. Can you use only three colours on these six states? Draw the graph that these six states make.



3 Sudoku: A graph colouring game in disguise

1. Complete this 4×4 sudoku board: Fill the squares with numbers 1,2,3,4 so that no number appears twice in a row, column, or *sub-square*.
2. Draw the graph that models this sudoku board as a graph colouring problem. Hint: It should have 16 vertices, and each vertex should have 7 neighbours.

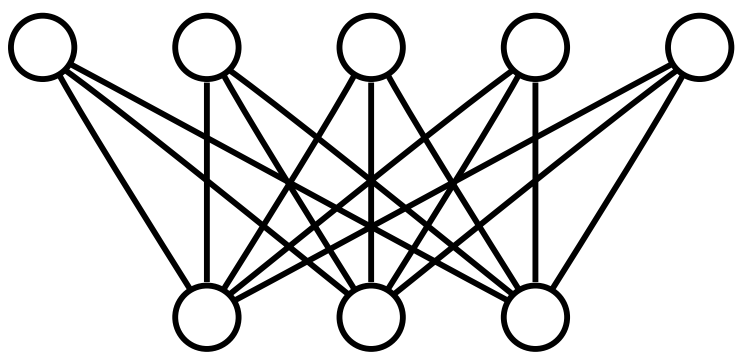
1		4	
			3
	1	3	

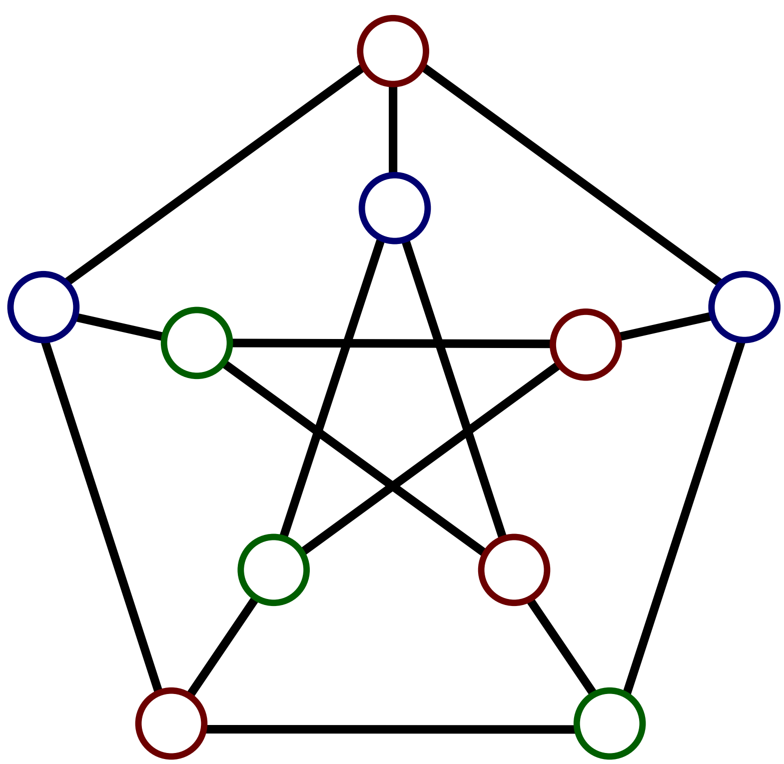
5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

4 Game coloring: Paint your opponent into a corner

- Player 1 and Player 2 alternate turns.
- There are k colours to choose from.
- At each turn, the player gives an *uncoloured* vertex an *available* colour.
- If all the vertices are coloured, **Player 1 wins**. If there is a stalemate (no available move) before all the vertices are coloured, **Player 2 wins**.

1. Try playing this colouring game on these graphs. What is the largest value of k for which Player 2 always has a winning strategy?
2. Explain why Player 2 can always win if $k = 2$.
3. Explain why Player 1 can always win if $k \geq 4$. This is pretty easy for the second graph, but the first graph is tougher.





5 Nim: Another combinatorial game

- There are three piles of stones, each containing between 0 and 5 stones.
- Player 1 and Player 2 alternate turns.
- At each turn, the player chooses a pile of stones (that is not empty) and removes *at least one* stone.
- The player to remove the last stone loses.

Play the game of Nim for the following instances. Can you see a pattern? Can you figure out a strategy? When can you be guaranteed a winning strategy?

3	4	5

1	1	1

1	2	1

1	2	4

2	2	2

4	4	4

1	1	5

3	2	1

3	3	3

0	2	2

0	3	3

5	2	2